DOCUMENT RESUME

ED 119 999 SE 020 420

AUTHOR Uram, Earl M.
TITLE The Case for Including Eulerian Kinematics in Undergraduate Dynamics.
PUB DATE Jun 75
NOTE 8p.; Paper presented at the annual conference of the American Society for Engineering Education (Colorado State University, Ft. Collins, June 16-19, 1975)

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage
DESCRIPTORS *College Science; Conference Reports; *Engineering Education; Higher Education; Instruction; *Mathematical Models; Science Education
IDENTIFIERS *Eulerian Kinematics

ABSTRACT

A Eulerian framework is proposed as an alternative to the Lagrangian framework usually used in undergraduate dynamics courses. An attempt to introduce Eulerian kinematics into a dynamics course is discussed. (LMH)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ED119999

3593
Event No.

AMERICAN SOCIETY FOR ENGINEERING EDUCATION

ANNUAL CONFERENCE, JUNE 16-19, 1975

COLORADO STATE UNIVERSITY

FT. COLLINS, COLORADO 80521

THE CASE FOR INCLUDING EULERIAN KINEMATICS IN UNDERGRADUATE DYNAMICS

Earl M. Uram
Sikorsky Professor of Engineering Mechanics
University of Bridgeport
Bridgeport, Connecticut 06622

Abstract

Undergraduate courses in Dynamics and virtually all undergraduate texts in the subject treat the study of the geometry of motion, i.e., Kinematics, in terms of a Lagrangian framework. It is rare indeed that a student completing the course realizes that another equivalent framework, the Eulerian framework, exists which is fundamental to and extensively used in later courses and subject areas that deal with continuous media; e.g., Fluid Mechanics, Heat Transfer, Thermodynamics, Gas Dynamics. The paper expands on these arguments and discusses the need for broader philosophical perspectives in teaching Dynamics. Experience with an attempt to introduce Eulerian kinematics along with examples into a Dynamics course, the amount of time alloted to the material and its impact upon subsequent courses is also discussed.

SE 020 420

THE CASE FOR INCLUDING EULERIAN KINEMATICS IN UNDERGRADUATE DYNAMICS

Introduction

This paper is intended to be an argumentative, critical essay concerning not only the rather small aspect of Dynamics dealing with the methods of representing the geometry of motion of dynamic systems but, more important, with the overall philosophy of approach and results of our present undergraduate treatment of the subject. To be sure, engineering students successfully completing Dynamics (for more than a decade, Vector Dynamics) courses certainly come away from them with the capability of analyzing dynamic mechanical systems. However, in my opinion, few, if any, are aware that there are other dynamic systems they certainly will be required to study and, perhaps later, work with as professionals. More important is the sad fact that they are totally unaware that the framework they have learned is not used for representing the system kinematics of these other systems. I am, of course, referring to those physical systems and subject areas for which the system itself or the material involved in it is considered a continuous media. As all of us are aware, such systems are encountered in Fluid Mechanics, Open Systems Thermodynamics, Elasticity and Convective Heat Transfer to mention several general subject areas without identifying specific systems.

For a time, it appeared that this situation might be rectified by the substantial interest in and development of courses in Continuum Mechanics. But, as you well know, the level of the courses introduced is such that in most engineering programs they might possibly be available to Juniors but most likely are offered as required or elective Senior and first year Graduate courses. It is remarkable that at least some of the unified philosophy or even the existence of the concept of a continuous media and where it is used have not found their way into the first Dynamics course. Yet, the first Dynamics course is universally assumed to be a prerequisite (even if not specifically identified

as such) for first courses in Fluid Mechanics and Thermodynamics.

Some Proposed Questions

Consider what the answers might be if you were to ask students having just completed a Vector Dynamics course any or some of the following or similar questions:

a) Do you know that there are two methods of representing motions i.e. velocities and accelerations, of dynamic systems, one proposed by J. L. Lagrange and another proposed by L. Euler? Which did you use in your Dynamics course? Under what circumstances or for what applications might it be more advantageous or preferrable to use the other?

b) Are you aware that the representation of velocities and accelerations in the dynamic systems you have just completed studying require that each mass element or particle must be identified at some initial time, t_o, by its then instantaneous position or space reference which thereafter remains invarient with time? That is, that you are essentially chasing after each particle or mass element once you have identified it? Is there another way?

c) Do you know of any physical system where the vector velocities and accelerations of its mass elements are or may be expressed in terms of both their position in space and time? If your answer is yes, why would you want to do this?

d) Suppose you were given a dynamic system for which it is known that the vector velocity, $\bar{V}$, of a mass element or particle is specified by both its spatial position, $\bar{r}$, at any instant with respect to a fixed origin in space and time, t; e.g., $\bar{V} = \bar{V}(\bar{r},t)$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. How would you determine the acceleration, $\bar{a}(\bar{r},t)$, for the particle at any point in its motion?

Of course, the questions are loaded and we all know what the students' answers might be. Consider also the fact that virtually all of the popular Dynamics texts that I have examined or used (thus virtually all courses using those texts) treat the dynamics of fluid jets or streams (and applications) as "variable systems of particles" in order to

correctly maintain use of the Lagrangian reference frame. Although the use of a control volume is implied and the mass conservation law is inferred in the analysis, neither is the former identified nor is the latter usually stated as a fundamentally required physical law for the system. Furthermore, the resulting impulse and momentum-based development and explanation of the phenomena is at least cumbersome and perhaps, for many students, difficult to digest. (These same arguments can be applied to the Conservation of Mass and First Law developments for open systems used by many thermodynamics texts where, to be sure, the control volume concept is well defined and explained. However, the use of an Eulerian reference frame is only implicit in these developments and not specifically defined, explained or utilized to obtain the final useful forms for these laws.)

Is it any wonder then that students are at least surprised if not confused when they see for the first time the convective acceleration terms that arise in the inertia term when Newton's law (momentum equation) is developed in Fluid Mechanics, or that they must at the same time learn, cope with and use both the calculus of vector fields and the physics of fluid and/or open thermodynamics systems, or that they conclude or believe that the dynamics of fluid and open thermal systems are different than that of rigid body or mechanical systems?

Why not, then, broaden the student's perspective of dynamic engineering systems by including at least discussion and illustrations of other than mechanical systems? Why not help prepare him to understand that the dynamical laws governing the behavior and response of all systems are the same but that for some, it is advantageous to use a different kinematical representation than for others and it is only the representation makes them <u>appear</u> different.

This can be accomplished if, along with certain concepts involved with the calculus of vector fields and some notions concerning continuous media, both Lagrangian and Eulerian kinematics are introduced side by side in dynamics.

Background Needed and Material Covered

Normally we can expect that second semester Sophomore engineering students have a knowledge of the calculus of functions involving more than one variable. If they have not been exposed to partial derivatives, it doesn't take much to provide them with sufficient knowledge to understand and handle them for the purpose at hand. They most likely will have to be introduced to the concepts and calculus of scalar and vector fields involving the "del" ($\bar{\nabla}$) vector operator with emphasis upon the physical meaning of the gradient of a scalar and the divergence of a vector. Ideally, it should be arranged for this material to be introduced in the third calculus course when vectors are discussed if it isn't already being done. The treatment of this material in Dynamics can then serve as a review with major emphasis placed upon physical interpretation. The curl of a vector should also be introduced just to complete the picture and for use in follow-on courses but need not be dwelled upon.

This is all that is needed as background to introduce the concepts of and comparatively discuss the differences between the Lagrangian and Eulerian methods for representing motions. This background material should take no more than one and one-half lectures with appropriate reading and homework assignments. Unfortunately, I know of no current Dynamics text that includes this material so that at this point instructor's notes and/or assigned supplementary reading material are necessary.

The concepts and comparison of Lagrangian and Eulerian methods for representing motions and the discussion of situations where each are advantageously and usually used can be covered in two lectures with supplementary reading and problem assignments.

Results of an Experiment

In the 1974 Spring Semester I taught a section of Dynamics containing a mix of fourteen Electrical, Computer and Mechanical Engineering majors. All of the M.E. majors were Sophomores while some of the other students were Juniors who had vector field theory exposure in a previous E.E. course (they later indicated they were glad to see the material

once more and that it found substantial use in subjects other than electrical fields). After discussing the nature and properties of vector differentiation, the concepts of scalar fields, directional derivative, gradient and properties of the del operator were introduced and their physical meaning discussed. This was then extended to include the notion of vector fields and the meaning of the divergence and curl of a vector.

A brief description of the nature of various specific illustrations representative of dynamic systems in the mechanical, fluid flow, thermal and electrical fields was done pointing out in which instance discrete mass elements or the notion of continuous media is used. With this as background, the fundamental philosophical difference between the methods of Lagrange and Euler for representing the time derivative of any mass element property and the mathematical formulation for doing so using each method was presented. I would recommend the presentation by S. Eskinazi* as one of the more lucid and concise treatments of this material. In addition, side by side illustrations showing that although the mathematical expressions for determining velocity and acceleration are quite different, the results are exactly the same.

This being the first trial at presenting this material to students at this level, it took slightly more time than suggested earlier in the paper. In the subsequent Fall semester the section of Fluid Mechanics I taught contained the same M.E. students and several of the E.E. students. In addition there were other M.E. students who had not taken Dynamics with me. Furthermore all of the M.E. students were concurrently taking the first Thermodynamics course with a different instructor. At the end of the semester during a frank discussion with the students regarding the material in the course, there was a definite response to my questions directed towards finding out whether introducing this material in Dynamics had any effect. There was a clear

*S. Eskinazi, "Principles of Fluid Mechanics", pp. 146-152, 2nd Edition Allyn and Bacon, Inc., 1968

feeling by those students having the previous exposure in Dynamics that they had an advantage in developing an understanding of associated material in both the Fluid Mechanics and Thermodynamics courses. They spent less time than the other students, as might be expected, studying to understand the vector calculus and kinematical relationships and more time with attempting to understand the Fluid and Thermal physics aspects of the courses. The other students also indicated the usual reaction to the convective acceleration terms. It should be noted that neither of the texts used in Fluid Mechanics and Thermodynamics that semester happened to have a comparative discussion of Eulerian and Lagrangian reference frames.

This one trial, of course, is insufficient evidence to support a suggestion for a drastic change. But I am proposing only a small change and I think the evidence is indicative that the change is most likely to have only very beneficial effects. Moreover, I submit that we have been at least short changing and perhaps misleading our students in Dynamics for too long. It certainly wouldn't take much on our part to make sure that the student has a much broader perspective of Dynamic systems in engineering than our current approach appears to afford him. This paper has been purposely provocative and I welcome your comments.